

1. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation which rotates vectors $\pi/3$ radians counterclockwise.

Which of the following matrices is the standard matrix of T ?

a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ c) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ d) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

2. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ the linear transformations defined by

$$T \left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \right) = \begin{bmatrix} x - y + z + 2w \\ 2x + y + 5z - w \\ x + 2y + 3z + w \end{bmatrix}$$

Which of the following statements about T is TRUE?

- a) T is onto but not one-to-one.
- b) T is one-to-one but not onto.
- c) T is one-to-one and onto.
- d) T neither one-to-one nor onto.

3. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation such that

$T \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $T \left(\begin{bmatrix} -1 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$. What is $T \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} -1 \\ 4 \end{bmatrix} \right)$?

a) $\begin{bmatrix} 14 \\ -8 \end{bmatrix}$ b) $\begin{bmatrix} 8 \\ -14 \end{bmatrix}$ c) $\begin{bmatrix} -8 \\ 14 \end{bmatrix}$ d) $\begin{bmatrix} -14 \\ 8 \end{bmatrix}$

4. Let A be a 5×4 matrix such that its row reduced echelon form has 4 pivot positions. Which of the following statements is/are TRUE?

- a) The linear transformation T defined by $T(x) = Ax$ is onto.
- b) $Ax = 0$ has a unique solution.
- c) Columns of A are linearly dependent.
- d) $Ax = b$ is consistent for every vector b in \mathbb{R}^5 .
- e) Row reduced echelon form of A has a zero row.

5. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation which rotates vectors $2\pi/3$ radians counterclockwise. Write down the standard matrix of T .

6. Let $T : R^2 \rightarrow R^3$ be the linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 3 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}.$$

a) Express each of the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ as a linear combination of the

vectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

b) Find $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.

c) Find the standard matrix of T .

7. Let $T : R^2 \rightarrow R^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}.$$

What is the standard matrix of T ?

8. Let T be the linear transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - z \\ x - y \\ 2x + y - z \\ y + z \end{bmatrix}$.

a) Find the standard matrix of T .

b) Find a vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ whose image under T is the vector $\begin{bmatrix} 2 \\ -2 \\ -4 \\ -8 \end{bmatrix}$.

c) Is the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ in the range of T ?

d) Is the vector $\begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$ in the range of T ?